523

NPSOR-92-005

NAVAL POSTGRADUATE SCHOOL

Monterey, California

UNITED STATES NAVY
NAVAL POSTGRADUATE SCHOOL

PYROTECHNIC DEVICE RELIABILITY

Lyn R. Whitaker
and
Michael Bailey

December 1991

Approved for public release; distribution is unlimited.

Prepared for:
Naval Weapons Support Center
Crane, IN 47522

NAVAL POSTGRADUATE SCHOOL,
MONTEREY, CALIFORNIA

This report was prepared for and funded by the Naval Weapons Support Center, Crane, Indiana.

This report was prepared by:

UNCLASSIFIED
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED	1b. RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY 2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	3. DISTRIBUTION /AVAILABILITY OF REPORT Approved for public release; distribution is unlimited.
4. PERFORMING ORGANIZATION REPORT NUMBER(S) NPSOR-92-005	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Naval Postgraduate School 6b. OFFICE SYMBOL (*If applicable*) OR	7a. NAME OF MONITORING ORGANIZATION Naval Postgraduate School
6c. ADDRESS (*City, State, and ZIP Code*) Monterey, CA 93943	7b. ADDRESS (*City, State, and ZIP Code*) Monterey, CA 93943-5006
8a. NAME OF FUNDING/SPONSORING ORGANIZATION Naval Weapons Support Center 8b. OFFICE SYMBOL (*If applicable*)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N4802991WXPM454
8c. ADDRESS (*City, State, and ZIP Code*) NWSC, Crane, IN 47522	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO. / PROJECT NO. / TASK NO. / WORK UNIT ACCESSION NO.

11. TITLE (*Include Security Classification*)
Pyrotechnic Device Reliability

12. PERSONAL AUTHOR(S)
Lyn R. Whitaker and Michael P. Bailey

13a. TYPE OF REPORT	13b. TIME COVERED	14. DATE OF REPORT (*Year, month day*)	15. PAGE COUNT
Technical Report	FROM______ TO______	December, 1991	25

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (*Continue on reverse if necessary and identify by block number*)
FIELD	GROUP	SUB-GROUP	Reliability, pyrotechnic devices

19. ABSTRACT (*Continue on reverse if necessary and identify by block number*)

This report looks at the problem of determining when to award bonuses for reliability improvement in pyrotechnic devices based on data gathered under a lot acceptance sampling plan.

20. DISTRIBUTION/AVAILABILITY OF ABSTRACT [x] UNCLASSIFIED/UNLIMITED [ ] SAME AS RPT. [ ] DTIC USERS	21. ABSTRACT SECURITY CLASSICIATION UNCLASSIFIED
22a. NAME OF RESPONSIBLE INDIVIDUAL L. Whitaker	22b. TELEPHONE (*Include Area Code*) (408) 646-3482 2c. OFFICE SYMBOL OR/Wh

DD Form 1473, JUN 86 *Previous editions are obsolete.* SECURITY CLASSIFICATION OF THIS PAGE
S/N 0102-LF-014-6603 UNCLASSIFIED

Pyrotechnic Device Reliability

Lyn R. Whitaker and Michael P. Bailey

Department of Operations Research

Naval Postgraduate School

Monterey, CA 93943

(408)646-3482

December 19, 1991

Abstract

This report looks at the problem of determining when to award bonuses for reliability improvement in pyrotechnic devices based on data gathered under a lot acceptance sampling plan.

1 Introduction

As government procurement contracts are now written, contractors have no incentive to improve the quality of the items they provide. To improve quality, the Naval Weapons Support Center, Crane, IN, has decided to implement a bonus system. If, as a result of the sampling inspection plan currently required for lot acceptance, there is evidence that the lot has a reliability that exceeds the minimum requirement for lot acceptance, then the contractor will be eligible for a bonus. The difficulty with implementing such a system of bonuses is that the sampling inspection plans are designed to provide clear cut criteria for lot acceptance or rejections. They do not readily provide a single measure of quality or reliability of the item being tested.

The first item for which a bonus system will be implemented is a pyrotechnic device. The sampling inspection plan for this device involves sampling items from a lot. subjecting them to a manufacturing test and exposing them to one of three environments. An attempt is then made to activate each device. Specifically, a lot of 500 to 1000 devices is accepted if all of the following criteria are met:

1. Of 20 items subjected to the manufacturing test. no more than 1 can fail to activate.
2. Of 20 items subjected to the joint manufacturing and temperature and humidity test no more than 1 can fail to activate.
3. Of 32 items subjected to the joint manufacturing and vibration test no more than 2 can fail to activate.
4. Of 20 items subjected to the joint manufacturing and altitude test no more than 1 can fail to activate.

The samples in tests 1-4 are distinct . thus a total of 92 items are tested.

Attempts have been made to estimate the reliability R of a device (here reliability is defined as the probability that an item would activate after the manufacturing test and exposure to all three environments) based on data from the sampling inspection plan ([4] and [5]) . The methods used to produce lower confidence bounds LCB for this reliability based on data from a lot which is just barely accepted have yeilded values that are much lower than observed in the field.

The most obvious reason for this discrepancy is that requiring successful activation after exposure to all environments is much more stringent than requiring a device off the shelf to activate in practice. An operational environment would not include all these adverse conditions. It is also likely that items which activate after exposure to one environment would be more likely to activate after exposure to other environments. Unfortunately the presence or absence of such dependence cannot be inferred from the data yielded by the current sampling inspection plan. Thus, the models that have been used to estimate the reliability have assumed independence of the events that a device activates after exposure to the three environments given that it would have activated after the manufacturing test. If there is positive dependence between these events then estimation under the assumption of independence yields reliabilities and LCB's that are lower than they would be if dependence were accounted for. It should be noted that even if exact LCB's can be found, if the modeling assumptions are changed to include dependence (where these new assumptions are supported by empirical evidence), and if the 'true' reliability could be defined and estimated from the available data, the LCB's could still be rather low for the simple reason that the sample sizes of the sampling inspection plan are too small to give a sharper lower confidence bound.

In Section 2, we find the maximum liklihood estimator (MLE) of R under the assumption of independence of the events that a device activates after exposure to the three environments given that it has passed the manufacturing test. In the process, we uncover a difficulty with estimating R under this assumption. Specifically, this procedure can yield lower estimates of R for samples which have fewer failures. To fix this, lower bounds for the MLE of R under the conditional independence assumption are derived and shown to be lower bounds for MLE's under weaker and more realistic assumptions. Approximate 95% LCB's for R and the MLE are provided explicitly for the 24 realizations of the sampling plan leading to lot acceptance. In Section 3 a direct approach to awarding bonuses is taken. It provides an alternative to using LCB's for decision making. Conclusions and recommendations are given in Section 4.

2 Estimating Reliability

Tests 2, 3, and 4 in the lot acceptance sampling plan involve trying to activate the device after it is subjected to the manufacturing test and one of the environmental tests. To distinguish between the outcome of these joint tests and what would have happened had the device been subjected to one of the tests alone, let E_1, E_2, E_3, and E_4 be the respective events that a device can activate after being subjected to just the manufacturing test, the temperature and humidity test, the vibration test, and the altitude test. Then, $R_1 \equiv P(E_1)$ and $R_i \equiv P(E_1 \cap E_i)$, $i = 2, 3, 4$ are the probabilities that a device activates after being subjected to tests 1, 2, 3, or 4, respectively. The reliability of a device is defined as

$$R = P(E_1 \cap E_2 \cap E_3 \cap E_4).$$

R will be estimated based on X_i, the number of devices which activate out of the n_i that are subjected to test i. The sampling plan uses different devices randomly chosen from the lot in question. Thus, it is reasonable to assume that $X_1, \ldots, X_4$ are independent and $X_i \sim Binomial(n_i, R_i)$, $i = 1, \ldots, 4$.

2.1 Maximum Likelihood Estimation

The likelihood of observing $X_1 = x_1, \ldots, X_4 = x_4$ is given as

$$L = \prod_{i=1}^{4} \binom{n_i}{x_i} R_i^{x_i} (1 - R_i)^{n_i - x_i}. \tag{1}$$

where $0 \leq R_1 \leq 1, 0 \leq R_i \leq R_1, i = 2,3,4$. For the sampling plan described in the introduction, there are 24 realizations of $X_1 \ldots, X_4$ that lead to lot acceptance. The maximum likelihood estimates (MLE's) $\hat{R}_i$ of R_i for these 24 realizations are given in Table 1.

Because the same device is never actually subjected to more than one of the environmental tests, the estimators $\hat{R}_i$ of R_i alone are not enough to construct an MLE for R. In the subsequent subsections, we describe an estimator $\hat{R}_{CI}$ for R which is based on the assumption of *conditional* independence. After examining the shortcomings of $\hat{R}_{CI}$, we provide a second estimate $\hat{R}_{LB}$ which requires no assumptions concerning dependencies between test outcomes, and which provides a lower bound for R.

2.2 Conditional Independence

Additional assumptions about the dependence between the events $E_1 \ldots, E_4$ are needed to estimate R based on the current sampling plan data. The most direct approach for estimating R based on this data is to assume that, given E_1, that E_2, E_3, E_4 are independent events, (see [4] and [5] for further discussion.) The MLE's in Table 1 are also the MLE's for R_i under the assumption of conditional independence. With this assumption, when $R_1 > 0$, we have

$$\begin{aligned}
R &= P(E_1 \cap E_2 \cap E_3 \cap E_4) \\
&= P(E_2 \cap E_3 \cap E_4 | E_1) P(E_1) \\
&= P(E_2|E_1) P(E_3|E_1) P(E_4|E_1) P(E_1) \\
&= \frac{R_2 R_3 R_4}{R_1^2}.
\end{aligned}$$

Thus the MLE $\hat{R}_{CI}$ of R under this assumption is given by

Table 1: MLEs for $R_1, \ldots, R_4$ computed for the 24 realizations of the sampling plan leading to lot acceptance, where (i, j, k, l) represents the number of failures in tests $1, \ldots, 4$.

(i, j, k, l)	$\hat{R}_1$	$\hat{R}_2$	$\hat{R}_3$	$\hat{R}_4$
(0000)	1.000	1.000	1.000	1.000
(0001)	1.000	1.000	1.000	0.950
(0010)	1.000	1.000	0.969	1.000
(0100)	1.000	0.950	1.000	1.000
(1000)	0.989	0.989	0.989	0.989
(0011)	1.000	1.000	0.969	0.950
(0101)	1.000	0.950	1.000	0.950
(0110)	1.000	0.960	0.969	1.000
(1001)	0.986	0.986	0.986	0.950
(1010)	0.983	0.983	0.969	0.983
(1100)	0.986	0.950	0.986	0.986
(0020)	1.000	1.000	0.938	1.000
(0111)	1.000	0.950	0.969	0.950
(1011)	0.975	0.975	0.969	0.950
(1101)	0.981	0.950	0.981	0.950
(1110)	0.975	0.950	0.969	0.975
(0021)	1.000	1.000	0.938	0.950
(0121)	1.000	0.950	0.938	1.000
(1020)	0.983	0.983	0.938	0.983
(1111)	0.962	0.950	0.961	0.950
(0121)	1.000	0.950	0.938	0.950
(1021)	0.975	0.975	0.938	0.950
(1120)	0.975	0.950	0.938	0.975
(1121)	0.950	0.950	0.938	0.950

$$\hat{R}_{CI} = \begin{cases} \frac{\hat{R}_2\hat{R}_3\hat{R}_4}{\hat{R}_1^2} & \hat{R}_1 > 0 \\ 0 & \hat{R}_1 = 0. \end{cases}$$

Realizations of $\hat{R}_{CI}$ for the 24 outcomes that lead to lot acceptance are given in Table 2.

From Table 2 it is clear that the assumption of conditional independence is unacceptable. Under conditional indepence, there are several cases where sampling plans resulting in fewer failures give smaller estimated value of R than with more failures. An example of this is the failure configuration (1, 1, 1, 0) with $\hat{R}_{CI} = .944$ compared to the case (0, 1, 1, 0) with $\hat{R}_{CI} = .920$. What happens in the (0, 1, 1, 0) case is that the conditional indepence assumption will attribute a disproportionate amount of the cause of failure to the lot's inability to pass environmental tests 2 and 3. Thus, the estimated probability of passing *all* tests is downgraded too much. This contradiction will lead us to dispose of the conditional independence assumption altogether. We will now make the results of this phenomenon formal.

Data from the lot acceptance sampling plan can be thought of as censored. For tests 2 - 4, it is unknown whether failure is due to the manufacturing test, the particular environmental test, or both. This lack of information, along with the conditional independence assumption, causes the apparent reversals in $\hat{R}_{CI}$ observed in Table 2. To see this, suppose that we could determine the cause of failure for a particular device which fails to activate. For test i, let

- $n_i - x_i$ be the total number of failures for test i;
- $x_{i,1}$ be the number of failures resulting from *only* the environmental exposure (these would have passed the simple manufacturing test);
- $x_{i,2}$ be the number of failures resulting from *only* the manufacturing test, (these would

Table 2: The MLE for R and an approximate 95% LCB computed for the 24 realizations of the sampling plan that lead to lot acceptance, where (i, j, k, l) represent the number of failures in tests 1-4 respectively

(i, j, k, l)	$\hat{R}_{CI}$	95% LCB
(0000)	1.000	1.000
(1000)	0.989	.900
(0010)	0.969	.906
(1010)	0.969	.855
(0001)	.950	.850
(0100)	0.950	.850
(1001)	0.950	.828
(1100)	0.950	.926
(1011)	0.944	.802
(1110)	0.944	.800
(1111)	0.939	.765
(0020)	0.938	.875
(1020)	0.937	.813
(1121)	0.937	.751
(0011)	0.920	.823
(0110)	0.920	.823
(1101)	0.920	.775
(1021)	0.913	.765
(1120)	0.913	.761
(0101)	0.902	.800
(0021)	0.891	.775
(0120)	0.891	.788
(0111)	0.874	.750
(0121)	0.846	.717

have passed the simple environmental test);

- $x_{i,3}$ be the number of failures resulting from *both* the manufacturing and the environmental tests.

The likelihood L is then proportional to

$$
\begin{aligned}
L &\propto P(E_1)^{x_1}(1-P(E_1))^{n_1-x_1} \\
&\prod_{i=2}^{4} P(E_1 \cap E_i)^{x_i} P(E_1 \cap E_i^c)^{x_{i,1}} P(E_1^c \cap E_i)^{x_{i,2}} P(E_1^c \cap E_i^c)^{x_{i,3}}.
\end{aligned}
$$

Let $n = \sum_{i=1}^{4} n_i$. Conditioning on the event E_1 gives

$$
\begin{aligned}
L &\propto P(E_1)^{\sum_{i=2}^{4} x_i + x_{i,1}}(1-P(E_1))^{n-\sum_{i=2}^{4} x_i + x_{i,1}} \\
&\prod_{i=2}^{4} P(E_i|E_1)^{x_i}(1-P(E_i|E_1))^{x_{i,1}} P(E_i|E_1^c)^{x_{i,2}}(1-P(E_i|E_1^c))^{x_{i,3}}.
\end{aligned}
$$

Therefore, the MLE's of interest are

$$
\begin{aligned}
\hat{P}(E_1) &= \frac{\sum_{i=1}^{4} x_i + x_{i,1}}{n} \\
\hat{P}(E_i|E_1) &= \frac{x_i}{x_i + x_{i,1}} \quad for\ i = 2, 3, 4.
\end{aligned}
$$

With these MLE's in hand, we can explain the behavior of the estimates in the conditional independence case. When E_2, E_3, E_4 are independent given E_1, we have

$$
\begin{aligned}
\hat{R}_{CI} &= \hat{P}(E_1) \prod_{i=2}^{4} \hat{P}(E_i|E_1) \\
&= \frac{\sum_{i=1}^{4} x_i + x_{i,1}}{n} \prod_{i=2}^{4} \frac{x_i}{x_i + x_{i,1}}.
\end{aligned}
$$

When the $x_{i,1}$ are not observed, the maximum likelihood approach assigns values

$$
\hat{x}_{i,1} = x_i \frac{\hat{P}(E_i^c \cap E_1)}{\hat{P}(E_i \cap E_1)}
$$

$$= x_i \frac{\hat{R}_1 - \hat{R}_i}{\hat{R}_i}$$

to $x_{i,1}$ where $\hat{R}_i$ are found by maximizing (1). This is what leads to the seemingly nonsensical ordering of reliabilities seen in table 2.1. The worst case was failure configuration (1, 1, 2, 1), for which we can compute

$$\begin{aligned}
\hat{x}_{2,1} = \hat{x}_{4,1} &= 0 \\
\hat{x}_{3,1} &= 30\left(\frac{.95 - .9375}{.9375}\right) \\
&\approx 0.01333.
\end{aligned}$$

Alternatively, configuration (0, 1, 2, 1) results in

$$\begin{aligned}
\hat{x}_{2,1} &= 19\left(\frac{1 - .95}{.95}\right) \\
&\approx 1 \\
\hat{x}_{3,1} &= 30\left(\frac{1 - .9375}{.9375}\right) \\
&\approx 2 \\
\hat{x}_{4,1} &= 19\left(\frac{1 - .95}{.95}\right) \\
&\approx 1
\end{aligned}$$

Thus, the second configuration, although being clearly superior to the first, results in assigning *blame* to the environments as the cause of failure. Since our definition of reliability is the joint probability of activating under all circumstances, the failure configurations which appear to point to the manufacturing test as the cause of most of the failures actually yield greater reliabilities.

Allowing the maximum likelihood procedure to assign values to the censored observations leads to an ordering of the estimates of R that are counterintuitive. In the next section, we will develop a bound for R which doesn't use conditional independence, and which produces an ordering of estimates of R which is more believable.

2.3 A Reasonable Lower Bound

An alternative approach is to note that no matter what the censored observations are, we still have

$$\begin{aligned} \hat{P}(E_1) &\geq \frac{\sum_{i=1}^{4} x_i}{n} \\ \hat{P}(E_i|E_1) &\geq \frac{x_i}{n_i} \quad i = 2, 3, 4. \end{aligned}$$

Thus, under the assumption of conditional independence

$$\hat{R}_{CI} \geq \left(\frac{\sum_{i=1}^{4} x_i}{n}\right) \prod_{i=2}^{4} \frac{x_i}{n_i}$$

Note that the lower bound of $\hat{R}_{CI}$,

$$\hat{R}_{LB} = \left(\frac{\sum_{i=1}^{4} x_i}{n}\right) \prod_{i=2}^{4} \frac{x_i}{n_i} \qquad (2)$$

is also equal to the MLE when there is as much dependence between $E_1 \ldots, E_4$ as is possible. This maximum dependence case arises if we assume that all of the failures observed are the result of BOTH the manfacturor's test and and environmental test, i.e. $x_{i,3} = n_i - x_i, i = 2, 3, 4$.

$\hat{R}_{LB}$ for the failure configurations leading to lot acceptance are shown in Table 3. There are two advantages to using $\hat{R}_{LB}$ rather than $\hat{R}_{CI}$ to estimate R. First, estimates $\hat{R}_{LB}$ are ordered in a reasonable way, failure configurations with more failures always have lower

Table 3: The lower bound for the MLE of R computed for the 24 realizations of the sampling plan leading to lot acceptance where (i, j, k, l) represent the number of failures in tests 1–4 respectively.

(i, j, k, l)	$\hat{R}_{LB}$
(0000)	1.000
(1000)	0.989
(0010)	0.958
(1010)	0.948
(0001)	0.940
(0100)	0.940
(1001)	0.938
(1100)	0.930
(0020)	0.917
(1020)	0.907
(0011)	0.900
(0110)	0.900
(1011)	0.890
(1110)	0.890
(0101)	0.883
(1101)	0.873
(0120)	0.862
(0021)	0.862
(1021)	0.852
(1120)	0.852
(0111)	0.846
(1111)	0.836
(0121)	0.809
(1121)	0.800

values of $\hat{R}_{LB}$. Second, $\hat{R}_{LB}$ provides a lower bound for R under much weaker assumptions about the dependence of the test outcomes.

Following [4], suppose that a device is more likely to activate after exposure to an environment if it has activated after exposure to a previous environment. Specifically, suppose

$$\begin{aligned} P(E_3|E_1 \cap E_2) &\geq P(E_3|E_1) \\ P(E_4|E_1 \cap E_2 \cap E_3) &\geq P(E_4|E_1), \end{aligned}$$

then

$$\begin{aligned} R &= P(E_1)P(E_2|E_1)P(E_3|E_1 \cap E_2)P(E_4|E_1 \cap E_2 \cap E_3) \\ &\geq P(E_1)P(E_2|E_1)P(E_3|E_1)P(E_4|E_1). \end{aligned}$$

Thus, $\hat{R}_{LB}$ given in (2) provides a lower bound for the MLE of R under these much more plausible assumptions.

2.4 Lower Confidence Bounds

Also given in Table 2 are approximate 95% lower confidence bounds found by bootstrapping [3]. For each of the 24 realizations under consideration, 1000 bootstrap samples were generated as follows. Let $\hat{r}_{i,k}$, $i = 1, \ldots, 4$, $k = 1, \ldots, 24$ represent the value of $\hat{R}_i$ computed for the k^{th} realization. Then, for the k^{th} realization, the bootstrap samples consist of generating 1000 independent realizations of $X_{1,k} \ldots, X_{4,k}$ where $X_{1,k} \ldots, X_{4,k}$ are independent and $X_{i,k} \sim Binomial(n_i, \hat{r}_{i,k})$. From each of the 1000 bootstrap samples, the MLE of R is computed, and the 95% lower confidence bound is taken as the 5^{th} percentile of the empirical distribution of these estimates of R.

The lower confidence bound for configuration $(1, 1, 2, 1)$ is slightly higher than the lower confidence bound of [4]. These lower confidence bounds are, however, approximations and are possibly biased. To see how these lower confidence bounds perform, it is a simple matter to compute the probabilities that they cover the true value of R under prescribed dependence assumptions. Returning to the conditional independence case, we can use (1) to compute the probability of any sampling outcome given values of $R_1 \ldots, R_4$. For simplicity, take $R_1 = \ldots = R_4$, so that, under conditional independence, $R = R_i$. For $R > 0.9$, the confidence intervals given by the lower confidence bounds in Table 2 ALWAYS cover R if there is at least one failure. Thus, the true level of confidence for these bounds is precisely the probability that at least one failure occurs during lot acceptance testing, $1 - \alpha = 1 - R^{92}$. ($n = 92$ is the number of tested units.) Thus, for $R = 0.99$, the confidence level for the lower bound is approximately 60%, and decreases to 37% when R increases to 0.995.

This strange effect is due to the design of the lot acceptance plan . The plan is designed to ensure with 95% confidence that a lot is accepted if $min(R_1, \ldots, R_4) \geq 0.99$. With higher reliabilities, there is a very good chance that there won't be any failures during the test, and we are powerless to produce an *accurate* 95% lower confidence bound for R.

In this section, we have attempted to produce an accurate point estimate and lower confidence bound for R, with the ultimate goal of establishing thresholds for assigning bonuses. In the next section, we reapproach the problem of assigning bonuses.

3 Alternative Formulation

In this section, we take a fresh approach to the problem of awarding bonuses. Our decision will be of the form, award a bonus if

$$\hat{R}(\mathbf{X}) \geq c$$

where $\hat{R}(\mathbf{X})$ is a statistic, possibly an estimate of reliablility, based on the number of devices that activate in tests 1-4, $\mathbf{X} = (X_1, \ldots, X_4)$. As we have stipulated in the previous section, for $\hat{R}(\mathbf{X})$ to be reasonable, it must be nondecreasing in each of its arguements, thus if $x_i \geq y_i$, $i = 1, \ldots, 4$ then $R(\mathbf{x}) \geq R(\mathbf{y})$ where $\mathbf{x} = (x_1, \ldots, x_4)$ and $\mathbf{y} = (y_1, \ldots, y_4)$. In the previous section, $\hat{R}_{LB}$ has this property.

The ultimate goal of the bonus program is to reward manufacturers who provide goods which are more useful in operations than the government-specified level of acceptability. Thus, the bonus rule will be based on the goal of rewarding all manufacturers for which $min(R_1, \ldots, R_4) \geq r$ with probability at least $1 - \alpha$. The decision rule which accomplishes this is to award bonuses if

$$\hat{R}(\mathbf{X}) \geq c^*$$

where c^* is the largest value of c for which

$$1 - \alpha \leq P(\hat{R}(\mathbf{X}) \geq c) \tag{3}$$

for $R_1 = \ldots = R_4 = r$.

As an example, take $\hat{R}(\mathbf{X})$ to assign numbers $1, \ldots, 24$ to the outcomes leading to lot acceptance as listed in Table 4. Table 4 gives the cumulative probabilities $P(\hat{R}(\mathbf{X}) \geq c)$ when $R_1 = \ldots = R_4 = r$ for different values of r. Thus, if bonuses are to be awarded to manufacturers for which $min(R_1, \ldots, R_4) \geq 0.995$ with 95% chance of identifying these vendors based on a lot acceptance sampling plan, then the threshold score $c^* = 17$. Here, bonuses are given to manufacturers which have one or zero total failures, or have $(0, 0, 1, 1)$, $(0, 1, 0, 1)$, or $(0, 1, 1, 0)$ failures. Note that in this example the values were arbitrarily assigned and don't take into account the fact that test 3 uses a larger sample of 32 while the other tests are based on a sample size of 20.

Depending on the environment that the device will be used in, the reliability is probably closer to a weighted average of $R_1, \ldots, R_4$. Because these weights differ for different types of pyrotechnic devices and the weights themselves must be estimated based on the mission the devices are used in, a simple and conservative alternative is to consider $R_{min} = min(R_1, \ldots, R_4)$. The minimum also has the attractive feature that it's MLE can be found from the lot acceptance data without assuming anything about the dependencies between $E_1, \ldots, E_4$. Table 5 gives the MLE $\hat{R}_{min} = min(\hat{R}_1, \ldots, \hat{R}_4)$ along with $P(\hat{R}_{min} \geq c)$ when $R_1 = \ldots = R_4 = r$ for various values of r.

To see that this rule acheives the desired goal, in otherwords that

$$1 - \alpha \leq P(\hat{R}(\mathbf{X}) \geq c^*)$$

for $\hat{R}_{min} \geq r$ where c^* is chosen as in (3), and $\hat{R}(\mathbf{X})$ is a nondecreasing function of $X_1, \ldots, X_4$, we need the notion of stochastic ordering. The random variable X is stochastically greater than the random variable Y, denoted $X \overset{st}{\geq} Y$, if $P(X \geq x) \geq P(Y \geq x)$ for all x. In the multivariate extension of this notion, the random vector $\mathbf{X}$ is stochastically greater than the random vector $\mathbf{Y}$ if

$$E[h(\mathbf{X})] \geq E[h(\mathbf{Y})]$$

for all real-valued functions h which are nondecreasing in their arguements for which the expectation exists.

Now let $X_1, \ldots, X_4$ be independent with $X_i \sim Binomial(n_i, R_i)$ and $Y_1, \ldots, Y_4$ be independent with $Y_i \sim Binomial(n_i, r)$, where $R_i \geq r$. Then $X_i \overset{st}{\geq} Y_i$ for $i = 1, \ldots, 4$ and by Theorem 4.13 of [1]

$$\mathbf{X} = (X_1, \ldots, X_4) \overset{st}{\geq} \mathbf{Y} = (Y_1, \ldots, Y_4).$$

Table 4: Cumulative probabilities for $\hat{R}(\mathbf{X})$ when $R_1 = r, \ldots, R_4 = r$, where (i, j, k, l) represent the number of failures in tests 1-4 respectively.

c	(i,j,k,l)	$P(\hat{R}(X) \geq c)$ R=.99	R=.995	R=.999
24	(0000)	0.401	0.634	0.913
23	(0001)	0.482	0.697	0.931
22	(0010)	0.610	0.800	0.960
21	(0100)	0.691	0.863	0.979
20	(1000)	0.772	0.926	0.997
19	(0011)	0.798	0.936	0.998
18	(0101)	0.814	0.943	0.998
17	(0110)	0.840	0.953	0.999
16	(1001)	0.856	0.959	1.000
15	(1010)	0.882	0.970	1.000
14	(1100)	0.899	0.976	1.000
13	(0020)	0.919	0.984	1.000
12	(0111)	0.924	0.985	1.000
11	(1011)	0.929	0.986	1.000
10	(1101)	0.932	0.987	1.000
9	(1110)	0.938	0.988	1.000
8	(0021)	0.942	0.989	1.000
7	(0121)	0.942	0.988	1.000
6	(1020)	0.947	0.989	1.000
5	(1111)	0.948	0.989	1.000
4	(0121)	0.948	0.989	1.000
3	(1021)	0.950	0.990	1.000
2	(1120)	0.950	0.990	1.000
1	(1121)	0.950	0.990	1.000

Table 5: Cumulative probabilities for $\hat{R}_{min}$ when $R_1 = r, \ldots R_4 = r$, where (i, j, k, l) represent the number of failures in tests 1-4 respectively.

		$P(\hat{R}_{min} \geq c)$		
c	(i, j, k, l)	R=.99	R=.995	R=.999
1.000	(0000)	0.401	0.634	0.913
0.989	(1000)	0.482	0.697	0.931
0.969	(0010)	0.636	0.809	0.961
	(1010)			
0.960	(0110)	0.667	0.819	0.962
0.950	(0001)	0.918	0.980	1.000
	(0100)			
	(0011)			
	(0101)			
	(1001)			
	(1100)			
	(0111)			
	(1011)			
	(1101)			
	(1110)			
0.938	(0020)	0.952	0.990	1.000
	(0021)			
	(0121)			
	(1020)			
	(1111)			
	(0121)			
	(1021)			
	(1121)			
	(1121)	0.950	0.990	1.000

This gives us $\hat{R}(\mathbf{X}) \overset{st}{\geq} \hat{R}(\mathbf{Y})$ so that

$$P(\hat{R}(\mathbf{X}) \geq c^*) \geq P(\hat{R}(\mathbf{Y}) \geq c^*) \geq 1 - \alpha.$$

More formally, the procedure to award bonuses tests the null hypothesis $H_o : R_1 \geq r, \ldots, R_4 \geq r$. Defining the null hypothesis in this way alows us to control the level of significance α, i.e. the probability of not awarding a bonus when one is deserved. The likelihood ratio test (e.g. [2]) for this problem is to award a bonus (i.e. fail to reject H_o) if $\hat{R}(\mathbf{X}) \geq c^*$ where $\hat{R}(\mathbf{X})$ is given by

$$-\sum [X_i ln(\frac{r}{\hat{R}_i}) + (n_i - X_i) ln(\frac{1-r}{1-\hat{R}_i})] I(\hat{R}_i \leq r)$$

where $I(A)$ is the indicator function of the set A.

4 Conclusion

There are several factors that make using acceptance test data to award bonuses challenging. They all stem from the fact that the data is gathered from a lot acceptance sampling plan which was not designed to estimate an overall reliability, nor was it designed to give good estimates of reliabilities when they are higher than the minimal lot acceptance standards.

To use the data from the lot acceptance sampling plan, the most reasonable approach seems to use $\hat{R}_{min}$, and its distribution for $R_1 = \ldots = R_4$ to decide whether to award a bonus.

There are several ways that this decision rule can be improved. These initiatives were mentioned earlier, and include

1. Take larger samples or sample sequentially. This will reduce the chances of awarding a bonus to underserving manufacturers.

2. Construct a better measure of overall reliability. A viable option is a weighted average

$$R = \sum_{i=1}^{4} w_i R_i$$

where the weights w_i represent the proportion of the devices in the field that are subjected to environment i. This would involve either constructing or estimating different weights for different types of devices.

3. Use $R = P(E_1 \cap E_2 \cap E_3 \cap E_4)$ as the measure of overall reliability, but collect data to estimate the interaction between the environmental tests. R could then be estimated using a loglinear model based on censored observations.

5 Aknowledgement

John Bowden suggested the form of the lower bound for the MLE. and the computations for Tables 1, 2 and 3 were done by Altan Ozkil, Lt. Turkish Army. We would also like to thank Bill Kemple for his careful reading of this report.

References

[1] Barlow, R. E. and Proschan, F. (1981) *Statistical Theory of Reliability and Life Testing*, To Begin With, Silver Spring MD.

[2] Bickel, P. J. and Doksum, K. J.(1977) Mathematical Statistics: Basic Ideas and Selected Topics, Holden-Day, Inc., San Francisco, CA.

[3] Efron, B. (1982) *The Jackknife, the Bootstrap and Other Resampling Plans*. CBMS-NSF Regional Conference Series in Applied Mathematics.

[4] Gleser. A. (1990) "Pyrotechnic Reliability" personal communication to Naval Weapons Support Center, Crane IN.

[5] Spencer, F. W. and Easterling, R. G. (1984) "Lower Confidence Bounds on System Reliability Using Componenet Data: The Maximus Methodology", Technical Report SAND84-1199C, Sandia National Laboratories.

INITIAL DISTRIBUTION LIST

1. Library (Code 52) 2
   Naval Postgraduate School
   Monterey, CA 93943-5000

2. Defense Technical Information Center 2
   Cameron Station
   Alexandria, VA 22314

3. Office of Research Administration (Code 81) 1
   Naval Postgraduate School
   Monterey, CA 93943-5000

4. Prof. Peter Purdue 1
   Code OR-Pd
   Naval Postgraduate School
   Monterey, CA 93943-5000

4. Technical Typist 1
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA 93943-5000

5. Prof. Lyn R. Whitaker 20
   Code OR-Wh
   Naval Postgraduate School
   Monterey, CA 93943-5000

5. Prof. Michael Bailey 20
   Code OR-Ba
   Naval Postgraduate School
   Monterey, CA 93943-5000